Method for analysing multi-path power flow transmissions

T Ciobotaru[1], D Frunzeti[2]*, I Rus[2], and L Jäntschi[2]
[1]Military Technical Academy, Bucharest, Romania
[2]Technical University of Cluj-Napoca, Cluj-Napoca, Romania

The manuscript was received on 27 May 2009 and was accepted after revision for publication on 16 December 2009.

DOI: 10.1243/09544054JEM1668

Abstract: This paper presents a method for fast calculation of the ratios and overall efficiency of epicyclic gearboxes applying a general method used for multi-path power flow transmissions. The method may be used as a first estimate for deriving conclusions regarding the manner in which the power is transmitted in multi-path power flow transmissions and has immediate application in tracking down of the situation of closed loop power circulation. In addition, the method presented has direct applicability for the analysis of epicyclic gearboxes, as well as for tracked vehicles' transmissions with multi-path power transmission, during the steering of the vehicle.

Keywords: epicyclic gear, epicyclic gearbox, power flows, efficiency

1 INTRODUCTION

Automatic transmissions are used increasingly frequently for cars, and have already become a standard for heavy tracked vehicles. The multi-path power flow allows the overall size of the transmission to be kept within acceptable limits because the power flow transmission uses two or more paths. An accurate assessment of overall efficiency of automatic transmissions must take into consideration the load and velocities of the external elements of the epicyclic gear mechanism [**1**].

Methods for kinematic analysis of epicyclic gearboxes have been developed using graphical approaches [**2**] and analytical approaches [**3**]; the analytical methods allow the use of computers to improve the efficiency of the calculations. These methods enable the overall gearbox ratios and the angular velocities of the external elements of the epicyclic gear mechanism (EGM) to be obtained.

A fast analysis of power flows is necessary to evaluate efficiency. Some graphical methods for analysing the kinematics of automatic transmissions and a method for evaluation of the power flows in two-path power flow transmissions were developed in references [**2**], [**3**], and [**4**]. Consequently, the loads of the gears can be determined.

A generalized formulation for automatic transmission planetary gear trains methodology, which required a complete static force analysis in order to evaluate the efficiency of the epicyclic gear trains, was proposed by Kahraman *et al.* [**5**].

The last necessary elements needed for the calculation of the overall efficiency of the gearbox are the efficiency of the EGMs. Various methods have been developed, which have in common the taking into consideration of the working parameters of the EGMs (torque and angular speed) [**6**, **7**].

The analysis of an epicyclic gearbox is intended to evaluate the gear ratio and the manner in which the power flows circulate. Eventually, the analysis must emphasize the evaluation of the overall efficiency of the epicyclic gearbox for every stage.

The current authors [**8**] have presented an exhaustive methodology that allows the calculation of the speeds and loads of the all of the elements in the transmission, including EGMs. The methodology allows the calculation of the overall efficiency of the transmission; it uses relations derived from the basic kinematics and torques balance of the EGMs and offers high accuracy. The price of this accuracy is the large amount of calculation and the high level of expertise needed for the equation formulation.

*Corresponding author: ARMA, Technical University of Cluj-Napoca, 103–105 Muncii Bvd, Cluj-Napoca, Cluj 400641, Romania.
email: danfrunzeti@yahoo.com

The actual paper presents a generalized methodology applied for assessing the power flow and the overall efficiency of multi-path power flow transmissions, with direct application to automatic transmissions. The proposed methodology proved to be useful and efficient as a first estimate, but also offers the potential for further developments to achieve better accuracy.

2 THE EPICYCLE GEAR MECHANISM

Epicyclic gearboxes consist of EGMs, brakes, and clutches. There is a large variety of EGMs, but the most frequently used have two or three degrees of freedom (DOF). Figure 1 presents the kinematic diagram of the most used EGM with two DOF; its structure consists of a sun gear, a planet gear and a carrier arm with satellite gears.

The general kinematic equation of the EGM is

$$\omega_1 - i^0_{1,2}\omega_2 - (1 - i^0_{1,2})\omega_0 = 0 \tag{1}$$

where $i^0_{1,2}$ is the ratio when the power is transmitted from element 1 (the sun gear) to element 2 (the planet gear), the element 0 (carrier arm) being fixed, and ω is the angular velocity of the element 1, 2, or 0 respectively.

Regarding the indices that indicate the elements of the EGM, the following rules apply (where n represents the total amount of EGMs within the gearbox):

(a) for sun gears: 1, 4, . . ., $3j - 2$, . . ., $3n - 2$;
(b) for planet gears: 2, 5, . . ., $3j - 1$, . . ., $3n - 1$;
(c) for carrier arm: 0, 3, . . ., $3j - 3$, . . ., $3n - 3$.

From equation (1) results the definition of the relative ratio $i^0_{1,2}$

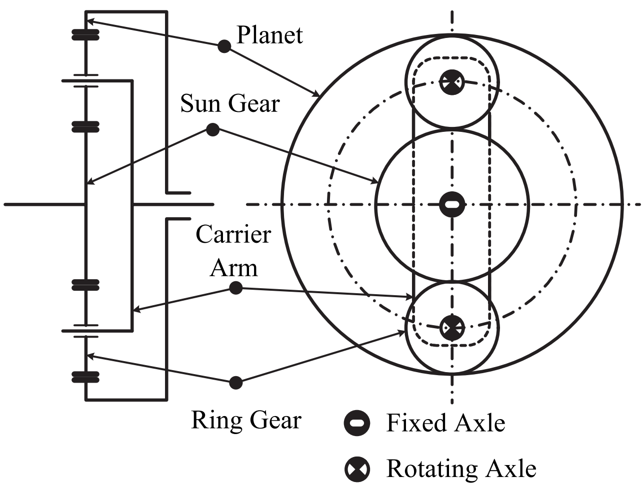

Fig. 1 Epicyclic gear mechanism

$$i^0_{1,2} = \left.\frac{\omega_1}{\omega_2}\right|_{\omega_0=0}. \tag{2}$$

By immobilizing the carrier arm, the EGM transforms into an ordinary gear mechanism, so the ratio $i^0_{1,2}$ could be calculated using the following relation

$$i^0_{1,2} = (-1)^{n_e}\, K \tag{3}$$

where n_e is the number of external meshes of the EGM, considering the existence of a single ring gear, and K represents the constant of the EGM, calculated with the relation

$$K = \frac{\text{Product of the teeth numbers of the driven gears}}{\text{Product of the teeth numbers of the driving gears}}$$

or, for the EGM presented in Fig. 1 and noting with Z the correspondent number of teeth, $K = Z_2/Z_1$.

For the particular case of the EGM presented in Fig. 1 results

$$i^0_{1,2} = -K \tag{4}$$

and the equation (1) takes the following aspect

$$\omega_1 + K\,\omega_2 - (1 + K)\,\omega_0 = 0 \tag{5}$$

Taking into consideration the relation (1), and imposing the condition $\omega_1 = 0$, the following relation results

$$\left.\frac{\omega_2}{\omega_0}\right|_{\omega_1=0} = -\frac{1 - i^0_{1,2}}{i^0_{1,2}} = i^1_{2,0} = \frac{1}{i^1_{0,2}} \tag{6}$$

Similarly, imposing the condition $\omega_2 = 0$, results in

$$\left.\frac{\omega_1}{\omega_0}\right|_{\omega_2=0} = 1 - i^0_{1,2} = i^2_{1,0} = \frac{1}{i^2_{0,1}} \tag{7}$$

The relations (6) and (7) allow any relative ratio to be expressed as a function of $i^0_{1,2}$. Consequently, in a generalized form, the equation (1) becomes

$$\omega_x - i^z_{x,y}\omega_y - (1 - i^z_{x,y})\omega_z = 0 \tag{8}$$

where $i^z_{x,y}$ represents a relative ratio between the element x and the element y, the element z being immobilized.

Regarding the torques, noted T, acting on the external elements of the EGM, the following equations apply

$$T_x + i^z_{x,y} T_y = 0 \tag{9}$$

$$T_x + T_y + T_z = 0 \tag{10}$$

For the specific case of the EGM considered above, the equations (9) and (10) become

$$T_1 - K\,T_2 = 0 \tag{11}$$

$$T_1 + T_2 + T_0 = 0 \tag{12}$$

An important conclusion from equations (11) and (12) is that if a torque acting on an external element becomes null, all other torques become null as well.

3 TRANSMISSIONS WITH TWO-PATH POWER FLOW

The usage of EGM with two DOF allows the power flows within structures to be transmitted through two or more paths. A multi-path transmission consists of several gear units and an EGM connected in order to constitute a structure. Figure 2 presents the simplest structure of a two-path power flow transmission.

The structure consists of the gears noted C and R respectively, and the EGM K. The EGM acts as a summing mechanism of the power flows transmitted through the gears C and R, respectively.

In order to calculate the overall ratio of the transmission, noted i_{ab}, it is necessary to express the angular velocities of the EGM external elements

$$\omega_x = i_{x,a} \cdot \omega_a \tag{13}$$

$$\omega_y = i_{y,a}\, \omega_a \tag{14}$$

$$\omega_z = \omega_b \tag{15}$$

Introducing the relations (13), (14), and (15) into equation (8), and taking into consideration the rule provided by the relation (7), finally results in

$$i_{a,b} = \frac{\omega_a}{\omega_b} = \frac{1 - i^z_{x,y}}{i_{xa} - i^z_{x,y}\, i_{ya}} = \frac{i^y_{x,z}}{i_{xa} - i^z_{x,y}\, i_{ya}} \tag{16}$$

After a few transformations, the relation (16) becomes

$$i_{a,b} = \frac{i_{ax} i^y_{x,z}}{1 - i_{ax}\, i^z_{x,y}\, i_{ya}} = \frac{i_{acb}}{1 - i_{acra}} \tag{17}$$

where $i_{acb} = i_{a,x}\, i^y_{x,z}$ represents the overall ratio of the path from a, through the gear C, to the output b, and $i_{acra} = i_{a,x}\, i^z_{x,y}\, i_{ya}$ represents the overall ratio of the closed loop starting from input shaft a, through the gear C, and the gear R (in the opposite direction) to the input shaft a.

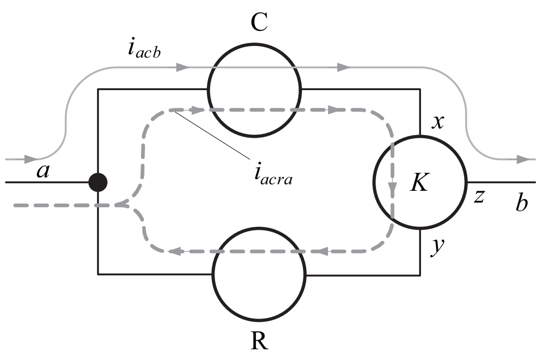

Fig. 2 The structure of two-path power flow transmission

Detailing the components of the overall ratios described above results in

$$i_{acb} = i_{ax}\, i^y_{x,z} = i_C\, i^y_{x,z} \tag{18}$$

$$i_{acra} = i_{ax}\, i^z_{x,y}\, i_{ya} = i_C\, i^z_{x,y}\, \frac{1}{i_R} \tag{19}$$

The relations (18), (19), and (17) allow fast calculation of the overall ratio of the transmission.

4 POWER FLOWS AND EFFICIENCY OF THE TWO-PATH POWER FLOW TRANSMISSION

For any type of transmission, the ratio is a rational function, F, having as variables the partial ratios of all the transmission components:

$$i_{ab} = \frac{\omega_a}{\omega_b} = \mathrm{F}\left(i_1, i_2, \ldots, i_q, \ldots, i_n\right) \tag{20}$$

where i_{ab} is the transmission overall ratio, ω_a is the angular velocity of the input shaft, ω_b is the angular velocity of the output shaft, a is the input shaft, b is the output shaft, and i_q is the ratio of the transmission component q.

The calculation of the ratio depends on the structure of the transmission components; for the particular case of the two-path power flow transmission the rationale function for ratio calculation is given by the relation (16).

The following relation defines the overall efficiency of the transmission

$$\eta_{a,b} = -\frac{\tilde{P}_b}{\tilde{P}_a} \tag{21}$$

where $\tilde{P}_a$ is the input power, $\tilde{P}_b$ is the output power, and the tilde symbol ($\sim$) applies to the real torques and powers in order to indicate the difference from ideal values.

Taking into consideration the definition of the power, the following relation results for the efficiency $\eta_{a,b}$

$$\eta_{a,b} = -\frac{\tilde{P}_b}{\tilde{P}_a} = -\frac{\tilde{T}_b\, \omega_b}{\tilde{T}_a\, \omega_a} = \frac{\tilde{i}_{a,b}}{i_{a,b}} \tag{22}$$

where $\tilde{T}_a$ is the input torque, $\tilde{T}_b$ is the output torque, and $\tilde{i}_{a,b}$ is the torque transformation ratio defined as

$$\tilde{i}_{a,b} = -\frac{\tilde{T}_b}{\tilde{T}_a} \tag{23}$$

The torques transformation ratio is expressed by the same function F, but having as variables the torque transformation ratios of the transmission units

$$\tilde{i}_{a,b} = \mathrm{F}\left(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_q, \ldots, \tilde{i}_n\right) \tag{24}$$

where

$$\tilde{i}_q = \eta_q^{u_q}\, i_q \tag{25}$$

in which u_q takes into consideration the direction of the power flow:

(a) $u_q = 1$ when the power flow has the same direction as the calculation of torque transformation ratio;
(b) $u_q = -1$ when the power flow has the opposite direction with respect to the calculation of torque transformation ratio.

Consequently, the exponent u_q is calculated as

$$u_q = \mathrm{sign}\tilde{P}_q \tag{26}$$

Taking into consideration the relations (20), (22), (24), and (25), the following relation results

$$\eta_{a\,b} = \frac{\mathrm{F}\left(\eta_1^{u_1}\, i_1, \eta_2^{u_2}\, i_2, \ldots, \eta_q^{u_q}\, i_q, \ldots, \eta_n^{u_n}\, i_n\right)}{\mathrm{F}\left(i_1, i_2, \ldots, i_q, \ldots, i_n\right)} \tag{27}$$

If P_τ represents the power losses of the transmission, then the overall efficiency may be expressed as

$$\eta_{a,b} = 1 - \frac{P_\tau}{\tilde{P}_a} \tag{28}$$

Meanwhile, the power loss, P_τ, is the sum of the power losses of the transmission components

$$P_\tau = \sum_{q=1}^{n} P_{\tau q} = \sum_{q=1}^{n} \tilde{P}_q\left(1 - \eta_q^{u_q}\right) \tag{29}$$

where n represents the number of components (ordinary gears and EGMs).

Introducing the relation (29) into the relation (28), the following relation results

$$\eta_{a,b} = 1 - \frac{1}{P_a} \sum_{q=1}^{n} \tilde{P}_q\left(1 - \eta_q^{u_q}\right) \tag{30}$$

The derivative of the overall efficiency with respect to the efficiency of the component q is

$$\frac{\partial \eta_{a,b}}{\partial \eta_q} = \frac{\partial}{\partial \eta_q}\left(\frac{\tilde{i}_{a,b}}{i_{a,b}}\right) = \frac{1}{i_{a,b}} \frac{\partial \tilde{i}_{a,b}}{\partial \eta_q} = \frac{1}{i_{a,b}} \frac{\partial \tilde{i}_{a,b}}{\partial \tilde{i}_q} \frac{\partial \tilde{i}_q}{\partial \eta_q} \tag{31}$$

and, taking into consideration that $\tilde{i}_q = \eta_q^{u_q}\, i_q$, the following relation results

$$\frac{\partial}{\partial \eta_q}\left(\frac{\tilde{i}_{a,b}}{i_{a,b}}\right) = \frac{i_q}{i_{a,b}} \frac{\partial \tilde{i}_{a,b}}{\partial \tilde{i}_q}\, u_q\, \eta_q^{u_q - 1} \tag{32}$$

The same derivative calculated taking into consideration the relation (30) gives

$$\begin{aligned}\frac{\partial \eta_{a,b}}{\partial \eta_q} &= \frac{\partial}{\partial \eta_q}\left[1 - \frac{1}{P_a}\sum_{q=1}^{n} \tilde{P}_q\left(1 - \eta_q^{u_q}\right)\right] \\ &= -\frac{1}{P_a}\left[\sum_{q=1}^{n} \frac{\partial \tilde{P}_q}{\partial \eta_q}\left(1 - \eta_q^{u_q}\right) - \tilde{P}_q\, u_q\, \eta_q^{u_q - 1}\right]\end{aligned} \tag{33}$$

Analysing the expression within the brackets, results in

$$\sum_{q=1}^{n} \frac{\partial \tilde{P}_q}{\partial \eta_q}\left(1 - \eta_q^{u_q}\right) \ll \tilde{P}_q\, u_q\, \eta_q^{u_q - 1}$$

and may be neglected. Consequently, from the relations (32) and (33) results

$$\frac{i_q}{i_{a,b}} \frac{\partial \tilde{i}_{a,b}}{\partial \tilde{i}_q}\, u_q\, \eta_q^{u_q - 1} \approx \frac{\tilde{P}_q}{P_a}\, u_q\, \eta_q^{u_q - 1}. \tag{34}$$

and because the ratio $i_{a,b}$ and the torque transformation ratio $\tilde{i}_{a,b}$ are expressed by the same function F, this results in

$$\frac{\partial \tilde{i}_{a,b}}{\partial \tilde{i}_q} = \frac{\partial i_{a,b}}{\partial i_q} \tag{35}$$

The input power $\tilde{P}_a$ does not depend on the existence of power losses; consequently

$$P_a = \tilde{P}_a \tag{36}$$

Finally, from equations (34), (35), and (36) the following relation results

$$\frac{\tilde{P}_q}{P_a} \approx \frac{i_q}{i_{a,b}} \frac{\partial i_{a,b}}{\partial i_q}$$

When the efficiency of the component q tends upwards to 1 (the ideal situation of no power losses), the power $\tilde{P}_q$ tends upwards to P_q, which then represents the power flow through the component q in the absence of the power losses

$$\lim_{\eta_q \to 1} \frac{\tilde{P}_q}{P_a} = \frac{P_q}{P_a} = \frac{i_q}{i_{a,b}} \frac{\partial i_{a,b}}{\partial i_q} \tag{37}$$

because in the relation (33)

$$\lim_{\eta_q \to 1}\left(1 - \eta_q^{u_q}\right) = 0$$

Defining the power coefficient β of the component q as

$$\beta_q = \frac{i_q}{i_{a,b}} \frac{\partial i_{a,b}}{\partial i_q} \tag{38}$$

the relation (37) becomes

$$P_q = \beta_q P_a \tag{39}$$

The relations (38) and (39) allow the fast calculation of the ideal power flows through the components of the transmission.

The existence of the power losses does not change the direction of the power flows; consequently

$$\mathrm{sign}\tilde{P}_q = \mathrm{sign}P_q$$

and from the relation (26) results

$$u_q = \mathrm{sign}P_q \tag{40}$$

From the relations (39), (38), and (40), the final relation which allows the calculation of the exponent u_q results in

$$u_q = \mathrm{sign}\frac{i_q}{i_{a,b}}\frac{\partial i_{a,b}}{\partial i_q} \tag{41}$$

In conclusion, for the fast calculation of the power flows and of the overall efficiency of the transmission, it is enough to know the functions F_j for each stage j of the gearbox.

For the particular situation of the epicyclic gearbox, the relation (27) becomes

$$\eta_{a,b} = \frac{\mathrm{F}\left(\eta_1^{u_1} K_1, \ldots, \eta_q^{u_q} K_q, \ldots, \eta_n^{u_n} K_n\right)}{\mathrm{F}\left(K_1, \ldots, K_q, \ldots, K_n\right)}, \tag{42}$$

where the exponents u_q are determined using the following relation

$$u_q = \mathrm{sign}\frac{K_q}{i_{a,b}}\frac{\partial i_{a,b}}{\partial K_q} \tag{43}$$

5 APPLICATION OF THE METHOD

In order to illustrate the usage of the above-described method, the kinematic diagram of a four-stage epicyclic gearbox is considered; the diagram is presented in Fig. 3 [**8**]. The analysis of the transmission investigated in reference [**8**] allows a comparison to be drawn between the amount from the calculations and from the results.

The gearbox consists of three EGMs, three brakes, and a clutch interconnected; the gearbox has two DOF. Figure 4 presents the equivalent block diagram drafted according to the kinematic diagram.

Regarding the indices that indicate the elements of the EGM, the following rules are introduced (where n represents the total amount of EGMs of the gearbox):

(a) for sun gears: 1, 4, ..., $3j-2$, ..., $3n-2$;
(b) for ring gears: 2, 5, ..., $3j-1$, ..., $3n-1$;
(c) for carrier arm: 0, 3, ..., $3j-3$, ..., $3n-3$.

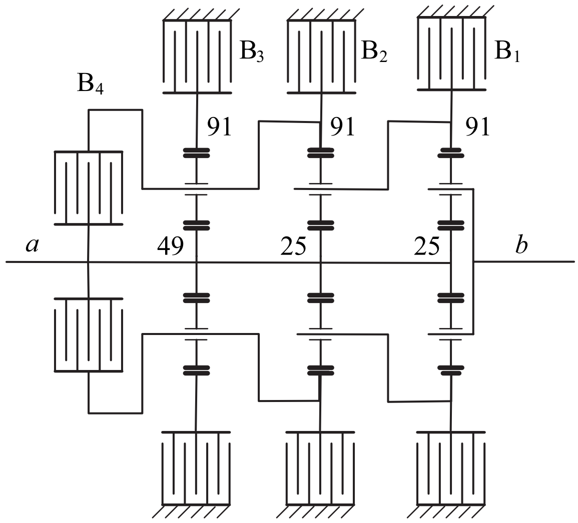

Fig. 3 Kinematic diagram of the four-stage epicyclic gearbox

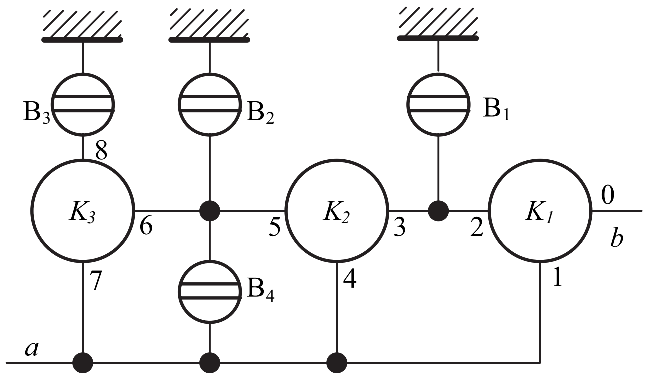

Fig. 4 Block diagram of the gearbox

Table 1 The sequence of brakes and clutch

Stage	B_1	B_2	B_3	B_4
1st	Operated			
2nd		Operated		
3rd			Operated	
4th				Operated

Each stage is realized by operating a single friction element. Table 1 presents the sequence of the brakes and clutch usage for obtaining the stages of the gearbox.

For the first stage, taking into consideration the EGM noted K_3, it may be observed that the torque T_8 is null because the brake B_3 is not operated. Consequently

$$T_6 = T_7 = 0$$

Following a similar line of argument, the following equation results

$$T_3 = T_4 = T_5 = 0$$

The entire power flow is transmitted through the EGM denoted K_1, used as demultiplier, with the element 2 fixed. The overall ratio for the first stage is

$$\left(i_{a,b}\right)_1 = i^2_{1,0}$$

and, after transformations, taking into consideration the relations (7) and (4)

$$i_{ab_1} = 1 + K_1 \tag{44}$$

The efficiency of the transmission is calculated using the relation (42)

$$\left(\eta_{a,b}\right)_1 = \frac{\left(\tilde{i}_{a,b}\right)_1}{\left(i_{a,b}\right)_1} = \frac{1 + \eta_1^{u_1} K_1}{1 + K_1}$$

The power flow transmission uses a unique path, therefore it results in $u_q = +1$. In order to perform the numerical calculations, an average value is taken into consideration for the efficiency of the EGM: $\eta = 0.9653$. Consequently, for $K_1 = 3.64$, the efficiency in the first stage results in

$$\left(\eta_{a,b}\right)_1 = \frac{\left(\tilde{i}_{a,b}\right)_1}{\left(i_{a,b}\right)_1} = \frac{1 + \eta K_1}{1 + K_1} = 0.9728$$

For the second stage, the coupling of the brake B2 determines that the power flows are transmitted through the EGMs denoted K_1 and K_2. Figure 5 presents the simplified block diagram of the gearbox in the second stage; this diagram is similar to the diagram presented in Fig. 2, with the difference that gear R is missing, so the ratio $i_R = 1$.

The ratio i_C can be easily calculated using the relation (7)

$$i_C = i^5_{4,3} = 1 - i^3_{4,5} = 1 + K_2 \tag{45}$$

The application of the relation (17) conducts to the following relation for the overall ratio in the second stage

$$\left(i_{a,b}\right)_2 = \frac{i_{acb}}{1 - i_{acra}} = \frac{i_C\, i^1_{2,0}}{1 - i_C\, i^0_{2,1}} \tag{46}$$

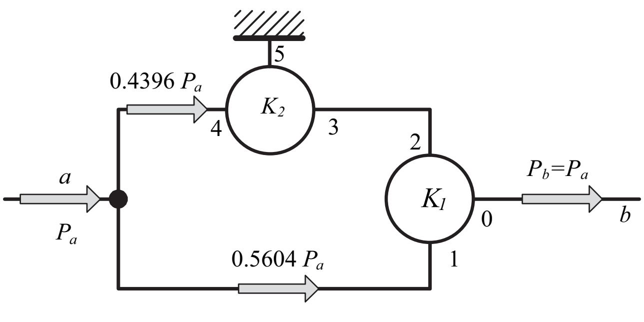

Fig. 5 The simplified block diagram for the gearbox in the second stage

Using the relations (6), (7), and (45), the final expressions for the gearbox ratio becomes

$$\begin{aligned}\left(i_{a,b}\right)_2 &= \frac{i_C\,(1 + K_1)}{K_1 + i_C} = \frac{(1 + K_1)\,(1 + K_2)}{1 + K_1 + K_2} \\ &= \frac{(1 + K_1)\,(1 + K_2)}{(1 + K_1)\,(1 + K_2) - K_1 K_2}\end{aligned} \tag{47}$$

The power flow transmitted through EGM K_2 is evaluated using the relation (38)

$$\beta_C = \frac{i_C}{\left(i_{a,b}\right)_2} \frac{\partial\left(i_{a,b}\right)_2}{\partial i_C} = \frac{K_1 + i_C}{1 + K_1} \frac{\partial}{\partial i_C}\left[\frac{i_C\,(1 + K_1)}{K_1 + i_C}\right]$$

The result of the above expression is

$$\beta_C = \frac{K_1}{K_1 + i_C} = \frac{K_1}{1 + K_1 + K_2} = 0.4396$$

The direction of the power flow is determined using the relation (41)

$$u_C = \operatorname{sign} \frac{i_C}{\left(i_{a,b}\right)_2} \frac{\partial\left(i_{a,b}\right)_2}{\partial i_C} = \operatorname{sign}\beta_C = +1$$

The power flows are indicated on the block diagram presented in Fig. 5, although it should be noted that these values do not take into consideration the power loss. In order to evaluate the power loss, the overall efficiency of the gearbox has to be evaluated. Consequently, assuming that all EGMs have the same efficiency ($\eta_i = \eta$, $i = 1, 2, 3$), the efficiency for this stage has the following expression

$$\left(\eta_{a,b}\right)_2 = \frac{\left(\tilde{i}_{a,b}\right)_2}{\left(i_{a,b}\right)_2} = \frac{(1 + \eta K_1)\,(1 + \eta K_2)\,(1 + K_1 + K_2)}{(1 + \eta K_1 + \eta K_2)(1 + K_1)\,(1 + K_2)}$$

resulting in

$$\left(\eta_{a,b}\right)_2 = 0.9761$$

It may be observed that, despite the use of two EGMs, the overall efficiency is greater compared with the efficiency of a single EGM, owing to the sharing of the power flows on two paths.

The third stage implies the work of all three EGMs. For a better understanding of the method applied, the elements area rearranged so the existence of two superimposed transmissions with two paths of power flow becomes obvious. Figure 6 presents the block diagram valid for the third stage of the gearbox. For the first two-path transmission structure, the EGM K_3 plays the role of transmission unit C, and has the ratio denoted i_{C1}; at the same time, this structure plays the role of transmission unit C for the overall gearbox, having the ratio denoted i_{C2}.

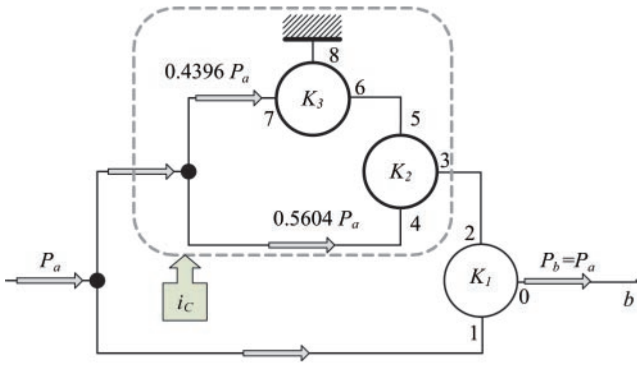

Fig. 6 The block diagram for the gearbox in the third stage

For the first structure, which is similar to the structure previously analysed, the overall ratio i_C is given by

$$i_{C2} = \frac{i_{C1}\, i_{7,6}^{8}}{1 - i_{C1}\, i_{5,4}^{3}} = \frac{i_{C1}\,(1+K_2)}{i_{C1}+K_2} = \frac{(1+K_3)\,(1+K_2)}{1+K_2+K_3} \tag{48}$$

Numerical calculation gives $i_{C2} = 2.0404$.

There is a similar result for the power flow through the EGM K_3

$$\beta_{C1} = \frac{i_{C1}}{i_{C2}}\,\frac{\partial i_{C2}}{\partial i_{C1}} = \frac{K_2 + i_{C1}}{1+K_2}\,\frac{\partial}{\partial i_C}\left[\frac{i_{C1}\,(1+K_2)}{K_2+i_{C1}}\right]$$

$$\beta_{C1} = \frac{K_3}{1+K_2+K_3} = 0.4396$$

The coefficient β_{C1} applies to the input power flow for the structure considered, and not to the gearbox input power.

The overall ratio of the gearbox in the third stage is calculated using the relation (17), considering the structure analysed above as a demultiplier with ratio i_{C2}

$$\left(i_{a,b}\right)_3 = \frac{i_{C2}\, i_{2,0}^{1}}{1 - i_{C2}\, i_{2,1}^{0}} = \frac{i_{C2}\,(1+K_1)}{i_{C2}+K_1} \tag{49}$$

Taking into consideration the relation (48), the final expression for the gearbox ratio is the following

$$\left(i_{a,b}\right)_3 = \frac{(1+K_1)\,(1+K_2)\,(1+K_3)}{(1+K_1)\,(1+K_2)\,(1+K_3) - K_1\,K_2\,K_3} = 1.667$$

The coefficient β_{C2} is calculated using the relation (49)

$$\beta_{C2} = \frac{i_{C2}}{\left(i_{a,b}\right)_3}\,\frac{\partial\left(i_{a,b}\right)_3}{\partial i_{C2}} = \frac{K_1 + i_{C2}}{1+K_1}\,\frac{\partial}{\partial i_{C2}}\left[\frac{i_{C2}\,(1+K_1)}{K_1+i_{C2}}\right]$$

resulting in

$$\beta_{C2} = \frac{1}{1 + \{[(1+K_2)\,(1+K_3)]/[K_1\,(1+K_2+K_3)]\}} = 0.6408$$

The coefficient of power flow through the EMG K_3 becomes

$$\beta_3 = \beta_{C1}\,\beta_{C2} = 0.2817$$

The results of the calculations are introduced into the block diagram presented in Fig. 6. Owing to the directions of the power flows, the exponent u_q take the following values

$$u_1 = u_2 = u_3 = +1$$

Consequently, the efficiency of the gearbox in the third stage becomes

$$\left(\eta_{a,b}\right)_3 = \frac{\left(\tilde{i}_{a,b}\right)_3}{\left(i_{a,b}\right)_3}$$

$$\left(\eta_{a,b}\right)_3 = \frac{(1+\eta K_1)\,(1+\eta K_2)\,(1+\eta K_3)}{(1+K_1)\,(1+K_2)\,(1+K_3)} = \frac{(1+K_1)\,(1+K_2)\,(1+K_3) - K_1\,K_2\,K_3}{(1+\eta K_1)\,(1+\eta K_2)\,(1+\eta K_3) - \eta^3\,K_1\,K_2\,K_3}$$

The numerical evaluation gives: $\left(\eta_{a,b}\right)_3 = 0.982$, this value being greater than the efficiency of an EGM working with fixed carrier arm.

6 CONCLUSIONS

The proposed methodology for analysing the epicyclic gearboxes has the following main advantages.

1. The methodology allows the fast calculation of the gear ratios and power flow for multi-path power flow transmissions with direct applicability for analysing an automatic transmission.
2. The methodology allows the fast evaluation of the power loss by calculating the overall efficiency of the gearbox.
3. The results of the analysis may serve as a first estimation for more accurate methodologies taking into consideration the influence of torque and speed on power loss of the EGMs.

Regarding the efficiency of the calculations, it is sufficient to mention that the usage of the methodology presented in reference [**8**] involves the solution, for each stage of the gearbox, of nine simultaneous equations derived from the kinematics, 12 simultaneous equations derived from the balance of the ideal torques, and 12 simultaneous equations derived from the balance of the real torques. The results of the calculations are the same, but the method presented above allows a faster process of calculations. In addition, the proposed method may generate adequate software application for the benefit of both undergraduate automotive students and gearbox design specialists.

REFERENCES

1 **Velardocchia, M., Bonisoli, E., Galvagno, E., Vigliani, A.,** and **Sorniotti, A.** Efficiency of epicyclic gears in automated manual transmission systems. In Proceedings of the 8th International Conference on *Engines for automobiles*, Capri, Naples, Italy, 2007, SAE paper 2007-24-0139.

2 **Oprean, I. M.** *Automatic transmissions for car*, 1999, pp. 20–91 (Printech Publishing House, Bucharest, RO).

3 **Karaivanov, A.** and **Popov, R.** Computer aided kinematic analysis of planetary gear trains of the 3k type. In Proceedings of the 3rd International Conference on *Manufacturing engineering (ICMEN)*, Chalkidiki, Greece, 2008, pp. 571–578.

4 **Filipoiu, I. D., Taille, R.,** and **Grecu, E.** Computing the efficiency of toothed gearings using power losses. In Proceedings of the 1st Europe–Asia Congress on *Mechatronics*, Besançon, France, 1996, Vol. 1, pp. 442–446.

5 **Kahraman, A., Ligata, H. K. K.,** and **Zini, D. M.** A kinematics and power flow analysis methodology for automatic transmission planetary gear trains. *Trans. ASME, J. Mech. Des.*, 2004, **126**(6), 1071–1081.

6 **Pennestri, E.** and **Freudenstein, F.** A systematic approach to power flow and static-force analysis in epicyclic spur-gear trains. *J. Mech. Des.*, 1993, **115**(3), 639–644.

7 **Pennestri, E.** and **Freudenstein, F.** The mechanical efficiency of epicyclic gear trains. *J. Mech. Des.*, 1993, **115**(3), 645–651.

8 **Ciobotaru, T., Frunzeti, D.,** and **Jäntschi, L.** A method for analyzing epicyclic gearboxes. *Int. J. Auto. Technol.*, 2010, **11**(2), 167–172. DOI: 10.1007/S12239-010-0022-4.

APPENDIX

Notation

a, b	input and output shafts, respectively
B_i	brake (clutch) i
C, R	generic gears of the two-path transmission
F_j	ratio of gearbox expressed as rational function with EGM constants as variables, calculated for stage j of the gearbox
$i_{a,b}$	ratio of the gearbox
$\tilde{i}_{a,b}$	torque transformation ratio defined as the ratio between the output torque, $\tilde{T}_b$, and the input torque, $\tilde{T}_a$
i_{acb}	overall ratio of the path from the input shaft a, through the gear C, to the output shaft b
i_{acra}	overall ratio of the closed loop starting from input shaft a, through the gear C, and the gear R (in the opposite direction), to the input shaft a
$i^z_{x,y}$	epicyclic gear mechanism ratio when the power is transmitted from element x to element y, the element z being fixed
K_i	constant of the EGM i, calculated as a ratio between the product of the teeth numbers of the driven gears and the product of the teeth numbers of the driving gears
n	total amount of EGMs of the gearbox
n_e	number of external meshes of the EGM
P_i	ideal (unaffected by power losses) power flow of EGM element i
$\tilde{P}_i$	real power flow (affected by power losses) of EGM element i
T_i	ideal torque (unaffected by power losses) of epicyclic gear element i
$\tilde{T}_i$	real torque (affected by power losses) of epicyclic gear element i
u_i	exponent of the efficiency of the planetary gear mechanism i
x, y, z	generic indices for the external elements of the EGM
Z_1	number of teeth of the sun gear
Z_2	number of teeth of the planet gear
η_i	efficiency of the EGM i, calculated for input on sun gear, output on planet gear, and fixed carrier arm
$(\eta_{a,b})_i$	gearbox overall efficiency for the stage i
ω_i	angular velocity of the element i