

A novel method for measuring the structural information content of networks

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Abstract

In this paper we first present a novel approach to determine the structural information content (graph entropy) of a network represented by an undirected and connected graph. Such entropic measures can be very important and useful to analyze and compare complex systems by means of networks. The novel graph entropy definition is based on local vertex functionals obtained by calculating j -spheres via the algorithm of Dijkstra. We state some lower and upper bounds of the defined graph entropy to estimate the structural information content for graph classes or explicitly given graphs. Second, we present a detailed example for calculating the graph entropies of a special graph class.

Keywords

Entropy; Graphs; Structural information; Structural information content; Topological entropy

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