

METHOD FOR ANALYZING EPICYCLIC GEARBOXES

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ABSTRACT—This paper presents a method for analyzing epicyclic gearboxes by evaluating the speeds, torques and power of the external elements in epicyclic gear mechanisms, as well as the total ratios of the gear box. The method is based on the equations that describe each epicyclic gear mechanism and rules that assign appropriate codes to the external elements. The method emphasizes how power flows are transmitted through the epicyclic gears, as well as power losses. Analysis of an epicyclic gear box is performed to illustrate the proposed method.

KEY WORDS : Epicyclic gear, Epicyclic gearbox

1. INTRODUCTION

Principles of Mechanisms (Willis, 1841) is widely considered to be the first publication dedicated solely to the field now called kinematics, and discusses analytical modeling of an epicyclic gear train for the first time in published literature. Later, *Analysis and Design of Mechanisms* (Lent, 1970) presents the methodology proposed by Willis for finding the rotational speeds of each branch of the epicyclic gear train in detail, along with specific methods for designing three- and four-gear trains. Further developments of the method were made by Tsai (2001), by using graph theory, and extending “the traditional concept of a lever representation of a planetary gear set to one that includes negative lever ratios” (Raghavan, 2007). These methods have been found to be suitable in a series of applications (Ashmore, 2006) and (Karaivanov and Popov, 2008).

Epicyclic gearboxes have been used as an alternative to other methods (Kim *et al.*, 2008b, 2008a) more frequently because of their advantages: compact design, automatic shifting, and increased control possibilities, for example.

The goal of analyzing an epicyclic gearbox is to evaluate the gear ratios, the power flow and the efficiency for every stage (Lee, 2007). This paper presents a methodology to analyze epicyclic gearboxes using elements of power flow theory. This methodology was established for the use a computer for efficient calculations, and to evaluate the symbolic expressions for gear ratios, torques and power flows for all the elements. This paper presents the methodology applied to a stationary working regime characterized by invariant speeds ($d\omega/dt=0$); consequently, the inertia of gears and shafts is neglected.

2. APPLICATION OF POWER FLOW THEORY

The epicyclic gear box consists of one or more epicyclic gear mechanisms (EGM), clutches and freewheels. Every element may be described using linear equations to calculate speeds, torques and powers (Fogarasy and Smith, 1995; Du and Zhang, 2007).

A power flow graph is a graphical representation of a system of linear equations. The power flow graph establishes the link between the input variables (speed, torque and power) and the output variables and emphasizes how the power flow is transmitted (Ryu *et al.*, 2009). Consequently, the power flow graph represents a visual tool to easily generate an epicyclic gearbox diagram, as well as an analysis tool.

The power transmitted by an element may be calculated using the following formula:

$$P = M \cdot \omega, \quad (1)$$

where P is power (W); M is torque (Nm); and ω is angular velocity (rad/sec).

The above formula indicates that the power is computed as the product of two factors: the load factor (torque) and kinematic factor (velocity). Both torque and velocity may be positive or negative; consequently, the power is positive (input or convergent power) or negative (output or emergent power).

The power flow graph uses the following basic elements:

- nodes that transform at least one factor (load or kinematic);
- arcs that transmit the power flow without changing the factors.

The graph elements obey the following two rules:

(1) For every node, the sum of convergent power balances the sum of emergent power and dissipated power; thus the total sum of power is zero:

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$$\sum_{i=1}^n M_i \cdot \omega_i + P_{\tau i} = 0, \tag{2}$$

where $P_{\tau i}$ represents the dissipated power in node i .

This rule is derived from the general law of energy conservation applied to the epicyclic gearbox structure.

(2) For every node, the sum of the torques is zero (the law of torque balance):

$$\sum_{i=1}^n M_i = 0. \tag{3}$$

There are several types of nodes used to describe the kinematic structure of epicyclic gearboxes. The most frequently used nodes are summarized in Table 1. Using the graphical symbols listed in the table below, any kinematic gearbox can be transformed into power flow graphs.

3. EPICYCLIC GEAR MECHANISM FUNDAMENTALS

The epicyclic gearbox consists of one or more epicyclic gear mechanism clutches and brakes. Its kinematic diagram is presented in Figure 1.

Consequently, the epicyclic gear consists of 3 elements with fixed axles (sun gear, ring gear and carrier arm) and one element with a rotating axle (planet gear). The epicyclic gear mechanism presented in Figure 1. Epicyclic gear. has 2 degrees of freedom (DOF), but there are mechanisms with more DOF, for instance, the Ravigneux mechanism.

According to the Willis principle, the equation describing the kinematics of EGM is as follows:

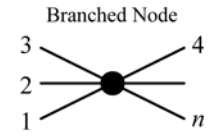

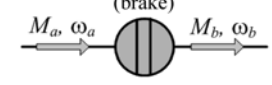
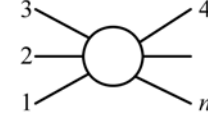
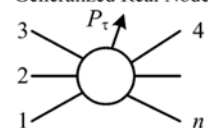
$$\omega_x - i_{x,y}^z \omega_y - (1 - i_{x,y}^z) \omega_z = 0, \tag{4}$$

where $i_{x,y}^z$ is the ratio when the power is transmitted from element x to element y and the axle of element z is fixed.

The indices that indicate the elements of EGM have the following rules (where n represents the total number of EGMs in the gearbox):

- for sun gears: 1, 4, ..., $3j - 2$, ..., $3n - 2$;

Table 1. Main types of nodes.

Node	Equations
<p>Branched Node</p> 	$\sum_{i=1}^n M_i = 0;$ $\omega_i = \omega_j;$ $i, j \in \{1, 2, \dots, n\}$
<p>Kinetic Node - Non operated clutch (brake)</p> 	$M_b = 0$
<p>Kinetic Node - Operated clutch (brake)</p> 	$M_b = M_a;$ $\omega_b = \omega_a$
<p>Generalized Ideal Node</p> 	$\sum_{i=1}^n M_i \cdot \omega_i = 0;$ $\sum_{i=1}^n M_i = 0$
<p>Generalized Real Node</p> 	$\sum_{i=1}^n M_i \cdot \omega_i + P_{\tau} = 0;$ $\sum_{i=1}^n M_i = 0$

- for ring gears: 2, 5, ..., $3j - 1$, ..., $3n - 1$;
- for the carrier arm: 0, 3, ..., $3j - 3$, ..., $3n - 3$.

Applying the index rules listed above, equation (4), in general, becomes the following:

$$\omega_{3j-2} - l_{3j-2,3j-1}^{3j-3} (1 - l_{3j-2,3j-1}^{3j-3}) \omega_{3j-3} = 0. \tag{5}$$

For a single EGM (in this case $j=1$), equation takes the following form:

$$\omega_1 - i_{1,2}^0 (1 - l_{1,2}^0) \omega_0 = 0. \tag{6}$$

If the carrier arm is fixed ($\omega_0=0$), EGM transforms into an ordinary gear mechanism with fixed axles; consequently, equation defines the ratio $l_{1,2}^0$:

$$l_{1,2}^0 = \frac{\omega_1}{\omega_2} \Big|_{\omega_0=0}. \tag{7}$$

For this ordinary mechanism, the EGM constant is defined as:

$$K = \frac{\text{Product of the teeth numbers of the driven gears}}{\text{Product of the teeth numbers of the driving gears}},$$

or for the EGM presented in Figure:

$$K = \frac{Z_2}{Z_1}. \tag{8}$$

In general, depending on the EGM structure:

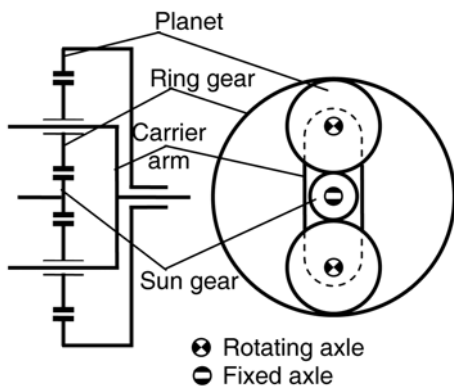


Figure 1. Epicyclic gear.

$$l_{1,2}^0 = (-1)^{n_e} \cdot K \quad (9)$$

or:

$$l_{3j-2,3j-1}^{3j-3} = (-1)^{n_{ej}} K_j \quad (10)$$

In conclusion, the kinematics of an EGM with 2 DOF is completely described by a single equation with 3 variables.

The distribution of torques among the EGM elements is given by the following system of equations:

$$\begin{cases} i_{x,y}^z M_x + M_y = 0 \\ M_x + M_y + M_z = 0 \end{cases} \quad (11)$$

From the first equation, the following relation results:

$$i_{x,y}^z = -\frac{M_y}{M_x} \quad (12)$$

For an ideal EGM (no power losses), equation (12) is similar to equation (7), but it has a different meaning. Equation (7) defines the kinematic ratio, and equation (12) defines the torque transformation ratio. For an ideal EGM, the two ratios (kinematic and torque transformations, respectively) are equal numerically.

For each external element of the EGM, the power is calculated using the following equation:

$$P_x = M_x \cdot \omega_x \quad (13)$$

An EGM with 2 DOF has the representation shown in Figure 1. It is assumed that there is an ideal meshing of gears, so there are no power losses.

In a real situation, the meshing of gears generates power losses, mainly due to the friction. The lost power is transformed into heat and is dissipated.

The existence of power loss modifies the torques but has no influence on the speeds of the external elements of the EGM. Consequently, equation (7) remains valid, but the real torque transformation ratio is given by the following equation:

$$\tilde{i}_{x,y}^z = -\frac{\tilde{M}_y}{\tilde{M}_x} \quad (14)$$

where the sign “~” (tilde) applies to the real torques and powers to indicate that they differ from their respective ideal values.

Considering only nodes with two external elements, the efficiency of the transmission is given by the following equation:

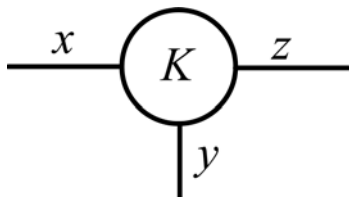


Figure 2. Power flow graph of the EGM with 2 DOF.

$$\eta_{xy}^u = -\frac{\tilde{P}_y}{\tilde{P}_x} = -\frac{\tilde{M}_y \cdot \omega_y}{\tilde{M}_x \cdot \omega_x} = \frac{\tilde{i}_{x,y}}{i_{x,y}^z}; u = \pm 1 \quad (15)$$

where the value of the exponent u takes into consideration the sense of the power flow. For a transmission including epicyclic gears, the value of the exponent u could be calculated using the following relation:

$$u_0 = \text{sign} \frac{i_{1,2}^0 \partial i_{a,b}}{i_{a,b} \partial i_{1,2}^0} = \text{sign} \frac{K \partial i_{a,b}}{i_{a,b} \partial K} \quad (16)$$

where indices a and b represent the input and output shafts, respectively.

4. ANALYSIS OF THE EPICYCLIC GEARBOX

Within an epicyclic gearbox, the power flows from input a to output b using one or more paths, according to the structure of the transmission.

Analyzing the epicyclic gearbox requires calculating the total ratios of the gearbox for each stage, the distribution of the power flows among the EGMs, and the overall efficiency for each stage. The proposed method calculates all these data in an efficient manner using commonly available software for solving algebra problems.

The method is best illustrated using a gearbox as an example; a kinematic diagram is presented in Figure 3.

The considered gearbox consists of 3 epicyclic gears with the constants $K_1 \dots K_3$, 3 brakes denoted by $B_1 \dots B_3$, and a clutch. Moreover, it can achieve 4 forward speeds. The input shaft is denoted by a , and the output shaft is denoted by b . This gearbox is part of the ZF 4 HP 250 transmission; the reverse is realized using an EGM as an inverter.

The sequence of the brakes and clutch usage is presented in Table 2.

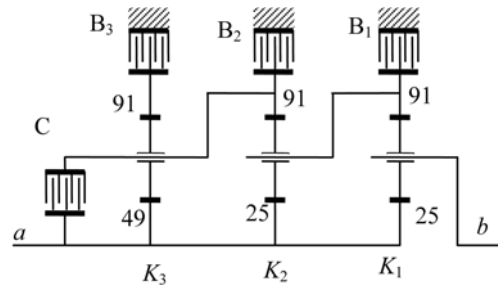


Figure 3. Kinematic diagram of the gearbox.

Table 2. Sequence of brakes and clutch.

Stage	B ₁	B ₂	B ₃	C
1st	operated			
2nd		operated		
3rd			operated	
4th				operated

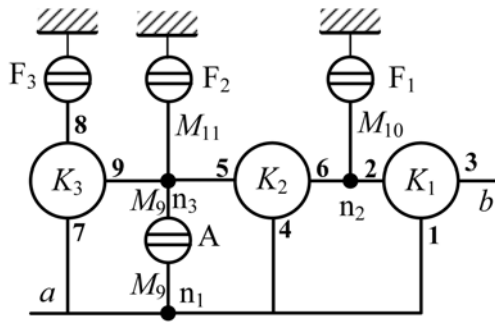


Figure 4. Power flow diagram of the epicyclic gearbox.

The equivalent power flow graph of the epicyclic gearbox is presented in Figure 4.

The epicyclic gearbox consists of 3 epicyclic gear mechanisms with constants K_1 , K_2 , and K_3 , respectively; 3 brakes, F_1 , F_2 and F_3 ; and the clutch A. The power flow diagram includes the branched nodes n_1, \dots, n_3 . The input shaft is labeled a , and the output shaft is labeled b .

The power flow diagram allows links between the external elements of the EGMs to be identified quickly, and facilitates calculation of the gearbox ratios.

The first step in analyzing the epicyclic gear box is calculating the gearbox ratios.

For each EGM, a single kinematic equation may be written. By rearranging the order of terms, the following equations are obtained:

$$(1 - i_{1,2}^0)\omega_0 - \omega_1 + i_{1,2}^0\omega_2 = 0; \quad (17)$$

$$(1 - i_{4,3}^3)\omega_3 - \omega_4 + i_{4,3}^3\omega_5 = 0; \quad (18)$$

$$(1 - i_{7,8}^6)\omega_6 - \omega_7 + i_{7,8}^6\omega_8 = 0. \quad (19)$$

The links between the external EGM's elements are described by the following equations:

$$\omega_1 - \omega_4 = 0; \quad (20)$$

$$\omega_1 - \omega_7 = 0; \quad (21)$$

$$\omega_2 - \omega_5 = 0; \quad (22)$$

$$\omega_5 - \omega_6 = 0. \quad (23)$$

The input element a is presumed to have a known speed: for convenience, it is assumed that:

$$\omega_1 = 1. \quad (24)$$

Consequently, the gear box ratio will be:

$$i_{a,b} = \frac{\omega_a}{\omega_b} = \frac{1}{\omega_8}. \quad (25)$$

Equations (17)~(24) are valid for each stage of the gearbox. A specific stage is obtained as a single brake or the clutch is activated according to the shifting diagram presented in Table 2. Consequently, the following equations may be written for each of the four stages:

- for the 1st stage:

$$\omega_2 = 0; \quad (26)$$

Table 3. Speeds for EGM's external elements.

	Gear 1	Gear 2	Gear 3
ω_0	$\frac{1}{1+K_1}$	$\frac{(1+K_1+K_2)}{\Pi_{1,2}}$	$\frac{\Pi_3 - K_1K_2K_3}{\Pi}$
ω_1	1	1	1
ω_2	0	$\frac{1}{1+K_2}$	$\frac{1+K_1+K_2}{\Pi_{2,3}}$
ω_3			
ω_4	1	1	1
ω_5	$-\frac{1}{K_2}$	0	$\frac{1}{1+K_3}$
ω_6			
ω_7	1	1	1
ω_8	$\frac{1+K_2+K_3}{K_2(1+K_3)}$	$\frac{1}{1+K_3}$	0
$i_{a,b}$	$1+K_1$	$\frac{\Pi_{1,2}}{(1+K_1+K_2)}$	$\frac{\Pi_3}{\Pi_3 - K_1K_2K_3}$
		$\Pi_3 = (1+K_1)(1+K_2)(1+K_3)$	
		$\Pi_{1,2} = (1+K_1)(1+K_2)$	
		$\Pi_{2,3} = (1+K_2)(1+K_3)$	

- for the 2nd stage:

$$\omega_5 = 0; \quad (27)$$

- for the 3rd stage:

$$\omega_8 = 0; \quad (28)$$

- for the 4th stage:

$$\omega_6 - \omega_7 = 0. \quad (29)$$

Adding one equation from relations (26)~(29) to the set of equations (17)~(24), a system of 9 simultaneous equations is obtained. Consequently, the nine variables $\omega_0, \dots, \omega_8$ can be calculated.

The calculation could be performed symbolically; thus, the ratio of the gearbox is expressed as a rational function whose variables are functions of the constants of the EGMs. Solving the systems of simultaneous equations for each stage, the following speeds were obtained.

In order to calculate the torques, the following equations are derived for each node:

- for EGM 1:

$$\begin{cases} M_1 + M_2 + M_0 = 0 \\ -K_1 M_1 + M_2 = 0 \end{cases}; \quad (30)$$

- for EGM 2:

$$\begin{cases} M_4 + M_5 + M_6 = 0 \\ -K_2 M_4 + M_5 = 0 \end{cases}; \quad (31)$$

- for EGM 3:

$$\begin{cases} M_7 + M_8 + M_6 = 0 \\ -K_3 M_7 + M_8 = 0 \end{cases}; \quad (32)$$

- for the branched node 1:

$$M_1 + M_4 + M_7 + M_9 = 1; \tag{33}$$

- for the branched node 2:

$$M_2 + M_3 + M_{10} = 0; \tag{34}$$

- for the branched node 3:

$$M_5 + M_6 + M_9 + M_{11} = 1. \tag{35}$$

The following restrictions apply for each stage respectively:

- for the 1st stage:

$$M_8 = 0; M_9 = 0; M_{11} = 0; \tag{36}$$

- for the 2nd stage:

$$M_8 = 0; M_9 = 0; M_{10} = 0; \tag{37}$$

- for the 3rd stage:

$$M_9 = 0; M_{10} = 0; M_{11} = 0; \tag{38}$$

- for the 4th stage:

$$M_8 = 0; M_{10} = 0; M_{11} = 0. \tag{39}$$

Equations (30)~(35) together with one of the restrictions (36)~(39) compose a system of simultaneous equations; for instance, for the first stage, the system is the following:

$$\begin{cases} M_1 + M_2 + M_0 = 0; & -K_1 M_1 + M_2 = 0 \\ M_4 + M_5 + M_6 = 0; & -K_2 M_4 + M_5 = 0 \\ M_7 + M_8 + M_9 = 0; & -K_3 M_7 + M_8 = 0 \\ M_1 + M_4 + M_7 + M_9 = 1 \\ M_2 + M_3 + M_{10} = 0 \\ M_5 + M_6 + M_9 + M_{11} = 0 \\ M_8 = 0; M_9 = 0; M_{11} = 0; \end{cases} \tag{40}$$

Taking into consideration equations (9), (15), and (16) and a real situation with power losses, the system of simultaneous equations (40) becomes:

$$\begin{cases} \tilde{M}_1 + \tilde{M}_2 + \tilde{M}_0 = 0; & -\eta_{01}^{u_1} K_1 \tilde{M}_1 + \tilde{M}_2 = 0 \\ \tilde{M}_4 + \tilde{M}_5 + \tilde{M}_6 = 0; & -\eta_{02}^{u_2} K_2 \tilde{M}_4 + \tilde{M}_5 = 0 \\ \tilde{M}_7 + \tilde{M}_8 + \tilde{M}_9 = 0; & -\eta_{03}^{u_3} K_3 \tilde{M}_7 + \tilde{M}_8 = 0 \\ \tilde{M}_1 + \tilde{M}_4 + \tilde{M}_7 + \tilde{M}_9 = 1 \\ \tilde{M}_2 + \tilde{M}_3 + \tilde{M}_{10} = 0 \\ \tilde{M}_5 + \tilde{M}_6 + \tilde{M}_9 + \tilde{M}_{11} = 0 \\ \tilde{M}_8 = 0; \tilde{M}_9 = 0; \tilde{M}_{11} = 0 \end{cases} \tag{41}$$

The exponents u_1 , u_2 , and u_3 can be calculated using the relations determined for gearbox ratios presented in Table 3.

To calculate the efficiency of the gearbox, equation (15) is applied to the overall structure of EGMs:

$$\eta_{ab} = \frac{\tilde{P}_b}{\tilde{P}_a} = \frac{\tilde{M}_b \cdot \omega_b}{\tilde{M}_a \cdot \omega_a} \cdot \tilde{i}_{a,b} \tag{42}$$

The gearbox ratio is expressed as a rational function

whose variables are the EGM constants:

$$i_{a,b_1} = F(K_1, K_2, K_3), i \in \{1, 2, 3\}. \tag{43}$$

In the presence of power losses, the torque transformation ratio is given by the same rational function but with different variables:

Table 4. Speed, torque and power flow calculation results.

	1st stage		2nd stage		3rd stage	
	No losses	Real	No losses	Real	No losses	Real
ω_0	0.21550		0.38451		0.60000	
ω_1	1		1		1	
ω_2	0		0.21551		0.49010	
ω_3						
ω_4	1		1		1	
ω_5	-0.27470		0		0.35000	
ω_6						
ω_7	1		1		1	
ω_8	0.62470		0.35000		0	
$i_{a,b}$	4.64000		2.60000		1.66700	
$M_0; \tilde{M}_0$	4.640	-4.514	-2.600	-2.538	-1.667	-1.637
$M_1; \tilde{M}_1$	1	1	0.560	0.562	0.359	0.363
$M_2; \tilde{M}_2$	3.64	3.514	2.040	1.975	1.308	1.274
$M_3; \tilde{M}_3$	0	0	-2.040	-1.975	-1.308	-1.274
$M_4; \tilde{M}_4$	0	0	0.440	0.438	0.282	0.282
$M_5; \tilde{M}_5$	0	0	1.600	1.538	1.026	0.992
$M_6; \tilde{M}_6$	0	0	0	0	-1.026	-0.992
$M_7; \tilde{M}_7$	0	0	0	0	0.359	0.355
$M_8; \tilde{M}_8$	0	0	0	0	0.667	0.637
$M_{10}; \tilde{M}_{10}$	3.640	3.514	0	0	0	0
$M_{11}; \tilde{M}_{11}$	0	0	1.600	1.538	0	0
$P_0; \tilde{P}_0$	-1	-0.973	-1	-0.976	-1	-0.982
$P_1; \tilde{P}_1$	1	1	0.560	0.562	0.359	0.363
$P_2; \tilde{P}_2$	0	0	0.440	0.426	0.641	0.624
$P_3; \tilde{P}_3$	0	0	-0.440	-0.426	-0.641	-0.624
$P_4; \tilde{P}_4$	0	0	0.440	0.438	0.282	0.2823
$P_5; \tilde{P}_5$	0	0	0	0	0.359	0.347
$P_6; \tilde{P}_6$	0	0	0	0	-0.359	-0.347
$P_7; \tilde{P}_7$	0	0	0	0	0.359	0.355
$P_8; \tilde{P}_8$	0	0	0	0	0	0
η_{ab}	1	0.973	1	0.976	1	0.982

$$\tilde{i}_{a,b_i} = F(\eta_1^{\mu_1} K_1, \eta_2^{\mu_2} K_2, \eta_3^{\mu_3} K_3), i \in \{1, 2, 3\}. \quad (44)$$

To perform the numerical calculations, an average value was adopted for the efficiency of the EGM: $\eta=0.9653$. The final numerical results are presented in Table 4.

For the first stage, the reduced power flow diagram is presented in Figure 5. It can be observed that only the epicyclic gear mechanism denoted K_1 contributes to the transmission of the power flow. The same conclusion could be formulated by analyzing the structure of the function derived for the first stage:

$$i_{a,b_1} = F_1(K_1) = 1 + K_1$$

Because the constant K_1 is the only variable, this EGM only transmits the power flow.

For the second stage, the ratio is calculated by:

$$i_{a,b_2} = F_2(K_1, K_2) = \frac{(1 + K_1)(1 + K_2)}{(1 + K_1)(1 + K_2) - K_1 K_2},$$

and the transmission of the power flow is realized by two EGMs; the same conclusion results from the diagram presented in Figure 6.

For this stage, the power flow is transmitted through two paths. It may be observed that the power flow is transmitted by each link in the same direction; in other words, there is no internal power flow within the closed loop.

For the third stage, brake F_3 is engaged. The power flow diagram for this stage is presented in Figure 7. It may be observed that, in the third stage, all epicyclic gear mechanisms contribute to power flow transmission. The input

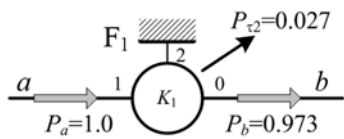


Figure 5. Power flow diagram for the 1st stage.

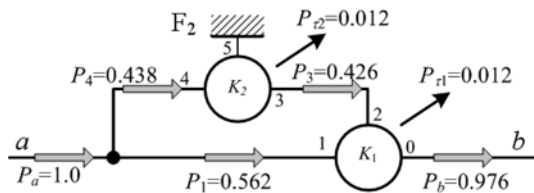


Figure 6. Power flow diagram for the 2nd stage.

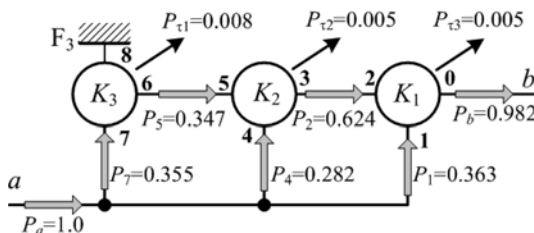


Figure 7. Power flow diagram for the 3rd stage.

power splits among the three EGMs, and the EGMs denoted K_2 and K_1 act as summing mechanisms.

5. CONCLUSIONS

The proposed method for analyzing the epicyclic gear boxes has the following main advantages:

- (1) It can be used to calculate the gear ratios, speeds, torques and powers for all paths;
- (2) It uses a computer for calculations, and the proposed algorithm can be used to generate adequate software;
- (3) It can evaluate the EGM constants if the numerical values of the ratios are known.

Additionally, the proposed method may generate adequate software applications that benefit both undergraduate automotive students and gearbox specialists.

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