A Study of Genetic Algorithm Evolution on the Lipophilicity of Polychlorinated Biphenyls

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The search for multivariate linear regression (MLR) in quantitative structure–property relationships (QSPR) is a hard problem, due to the dimension of the entire search space. A genetic algorithm (GA) was developed and assessed, to select proper descriptors for predicting the octan-1-ol/H₂O partition coefficient of polychlorinated biphenyls. The GA was implemented as a Windows based FreePascal application with MySQL connectivity for fetching the data. An outcome study based on 30 runs was done keeping all parameters constant: sample size, 8; number of variables in the MLR, 2; adaptation-imposed requirements; maximum number of generations, 1000; selection strategy, proportional; probability of mutation, 0.05; number of genes implied in mutation, 2; optimization parameter, r²; optimization score, minimum in sample; and optimization objective, maximum. The results revealed that the number of evolutions followed the Poisson distribution with the sample size as parameter. The average of the determination coefficient is higher than 98% of the determination coefficient obtained through complete search, and follows the Gaussian distribution. The correlation coefficients obtained by the best performing GA-MLR models proved not to be statistically different from the correlation coefficient of the QSPR model obtained by complete search.

Introduction. – Genetic algorithms (GAs) are derived from observations of natural phenomena and simulations of the artificial selection of organisms with multiple loci controlling a measurable trait [1][2]. GAs have evolved into complex and strong informatics tools able to deal with hard problems of decision, classification, optimization, and simulation [3]. GAs have also been used in drug design [4–7].

The structure–property/activity relationships (SP/ARs) establish functional links between the structure of chemical compounds and the associated physical and chemical properties (SPRs) or the biological activities (SARs) [8]. A huge variety of potential descriptors is available nowadays [9]. The computer is used to reduce the dimensionality of the descriptor space, to select those descriptors that have the highest contribution to the activity/property [10–12]. A series of techniques are used to select a subset of the most relevant descriptors: cluster analysis [13][14], principal component analysis [15], discriminate analysis [16][17], multiple linear regressions [18], partial least squares regression [19], factor analysis [20], GAs [21], machine learning [5][22], self-organizing-maps [23][24], and neural networks [25].

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The molecular descriptors family (MDF) approach [26] has been introduced and proved its abilities to identify the structure–activity/property relationships of several classes of compounds [27–29]. The aim of the present paper was to develop, implement, and assess the performances of a GA used to select the MDF subset with the highest explanation capacity for the octan-1-ol/H₂O partition coefficients of a sample of polychlorinated biphenyls (PCBs).

Results and Discussion. – The 60 descriptors presented in Table 1 were identified as being used by the genetic algorithm-based multivariate linear regression (GA-MLR) models with a high determination coefficient.

Table 1. Descriptors of the Molecular Descriptors Family Selected by Genetic Algorithms

<table>
<thead>
<tr>
<th>Type</th>
<th>Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical</td>
<td>IhMrFMg, isDdTcGg, iADrVGGg, IFndIcGg, IBdmlHg, isDRKHg, isDddHg, iamdIMg,</td>
</tr>
<tr>
<td></td>
<td>IHmmkHg, ismmFEg, iHPRSMGg, iHDriMGg, isMsVGGg, isMDFGg, IbMrVGGg, iammkEGg,</td>
</tr>
<tr>
<td></td>
<td>ismmkMGg, ISPmjGg, iHDrkGg, IbmrWGg, ISDrnGg, IaDdQEG, IbMdlHg, IbMdqGg, IbMdoMg,</td>
</tr>
<tr>
<td></td>
<td>IFMnkEg, lBnmmHg, iiMmHg, IBMdlHg, lBMmlHg, iimnSHg, lMddQEG, lMdlEG</td>
</tr>
<tr>
<td>Topological</td>
<td>INPRLcT, ISPRWCcT, INPRKGT, iAmDSET, innMlHT, IMPdOHcT, IHmmkHT, lMMrFGT, iSPRcET,</td>
</tr>
<tr>
<td></td>
<td>lSmREcT, ABMMPcT, iBMrmfHcT, IBPrODcT, inMRjGt, IbDrkGT, innRvGt, ISPDrSt, ImDrRGcT,</td>
</tr>
<tr>
<td></td>
<td>iSPDFcT, ihmDFM, lBmdoMt, IaDDDCt, iSDRFEt, ImMDjGT, ImPDKGT, ImMMKcT</td>
</tr>
</tbody>
</table>

A summary of the performances obtained in each run, expressed as the minimum and maximum values of the determination coefficient, the minimum and maximum values of the sum of residuals in the estimate, and the generation (out of 1000) in which the maximum value of the correlation coefficient was obtained, are presented in Table 2.

The analysis of the alive regressions in cultivar revealed the followings:
- The minimum value varied from 2 (Runs 9, 11, and 15) to 66 (Run 26) with a mean of 20 (95% CI (15–26)) and a standard error of 2.75.
- The most frequent minimum number of alive regressions was 12 (Runs 2, 19, 23, and 25).
- The maximum number of alive regressions varied from 80 (Run 11) to 288 (Run 22) with a mean of 172 (95% CI (151–194)) and a standard error of 10.48.
- The most frequent value of the maximum number of alive regressions was 128 (Runs 2, 3, 24, and 30).
- The range between the maximum and minimum of alive regressions on a run varied from 78 (Run 11) to 244 (Run 22) with a mean of 152 (95% CI (133–171)) and a standard error of 9.41. The minimum value of the range was obtained in a run, in which both the minimum and maximum values of alive regressions in cultivar were minimum (Run 11). The maximum range was obtained in Run 22, in which the value of the number of alive regressions in cultivar was maximum.
The evolution of the GA in terms of determination coefficients in the runs for the minimum and maximum numbers of alive regressions in cultivar is presented in Fig. 1. Note that the minimum number of alive regressions usually comes from the first generation (correlates with the initial solution), while the maximum number of alive regressions comes from the most populated generation.

As expected, it was observed that when the GA was run 30 times, 30 different QSPR models were identified for the investigated PCBs (see Table 2). The GA-MLR model...
with the highest determination coefficient was obtained in Run 7. The characteristics of this model are:

\[
\hat{Y}_{GA-MLR} = 1.139(\pm 0.917) - 0.237(\pm 0.079) \cdot \text{ImPdQHt} + 0.217(\pm 0.019) \cdot \text{iAmdlMg}
\]

\[
r = 0.9384 \ (95\% \ CI \ (0.9196 - 0.9528))
\]

\[
r^2 = 0.8807, \ s_{\text{est}} = 0.29, \ F_{\text{est}} (p_{\text{est}}) = 749 \ (1.93 \cdot 10^{-91})
\]

\[
r^2_{\text{cv-loo}} = 0.8763, \ s_{\text{loo}} = 0.29, \ F_{\text{pred}} (p_{\text{pred}}) = 719 \ (2.17 \cdot 10^{-93})
\]

where \( \hat{Y}_{GA-MLR} \) is the estimated octan-1-ol/H\(_2\)O partition coefficient (GA-MLR), \( \text{ImPdQHt} \) and \( \text{iAmdlMg} \) are molecular descriptors, and \( r \) is the correlation coefficient, \( r^2 \) the determination coefficient, \( s_{\text{est}} \) the standard error of estimate, \( F_{\text{est}} \) the F-value and associated probability, \( r^2_{\text{cv-loo}} \) the cross-validation leave-one-out score, \( s_{\text{loo}} \) the standard error of predicted, and \( F_{\text{pred}} \) the F-value and associated significance in the leave-one-out analysis, respectively. The descriptors used in this model and the associated residuals are available as Supplementary Material\(^1\). The graphical representations of the best performing GA-MLR search vs. the complete MLR search is presented in Fig. 2.

In terms of GA scores, the optimum scores were obtained in the run, in which the highest determination coefficient was obtained (see Table 2). Thus, in Run 7, three out of four GA scores, represented by the minimum value of the sum of residuals in estimate, the maximum value of the determination coefficient, and the minimum value of the determination or undetermination entropy, were optimum. The analysis of the geometric mean of evolution revealed that the minimum value of \(-1166.71\) was obtained in Run 25, while the maximum value of 975.79 was obtained in Run 15 (the

\(^1\) Supplementary Material may be obtained upon request from the authors.)
The absolute minimum value of 21.34 was obtained in Run 9, the absolute maximum value of 1166.71 was obtained in Run 25. The value of the geometric mean of evolution seems to be the score that is not related with the other imposed scores in the evaluation of GA performances.

Three criteria were used to assess the GA performance: i) the evolution (number of generations in which the determination coefficient improved), ii) the generation for which the determination coefficient has not changed until the end, and iii) the determination coefficient. The main statistical characteristics of these criteria are presented in Table 3.

The following were revealed to be true, when the distribution of the investigated criteria was analyzed:
- The evolution distribution proved to be Poisson (number of categories = 12, \( \lambda = 8.00 \), df (degree of freedom) = 2, Kolmogorov–Smirnov statistics = 0.136, Chi-square statistics = 1.446, and \( p = 0.485 \)).
- The maximum frequency distribution (for ten classes) of the number of generations in which the maximum value was obtained was observed in the following classes:

| Table 3. Statistical Characteristics of the Genetic Algorithm Assessment |
|---------------------------------|-------|-------|-------|-------|-------|
| Criterion | Minimum | Maximum | Mean | Median | Mode |
| Evol\(^a\)) | 3 | 15 | 8 | 7.5 | 6 |
| Gen\(_{\text{max}}\)\(^b\)) | 5 | 961 | 516 | 568 | 908 |
| \( r^2 \)\(^c\)) | 0.8684 | 0.8807 | 0.8759 | 0.8760 | n.a.\(^d\)) |

\(^a\)) Evol = Evolution (number of iteration in which \( r^2 \) improved). \(^b\)) Gen\(_{\text{max}}\) = Generation in which \( r^2 \) reaches the maximum value. \(^c\)) \( r^2 \) = Determination coefficient. \(^d\)) n.a. = Not available.
(100–200] and (900–1000] (see Fig. 3). This dispersion could not be assigned to any known frequency distribution.

- The determination coefficient distribution followed a normal distribution (number of categories, 15; Kolmogorov–Smirnov statistics, 0.087; Chi-square statistics, 0.306; \( p = 0.580 \)).

The convergence to the optimum solution sustained the ability of the GA to select the best two MDF members able to explain the structure–property relationships for PCBs. The GA speed (how many generations are needed to obtain the optimum solution) varied widely (from 5 to 961, see Table 3). The generation distribution for which the maximum value of the determination coefficient is obtained could be explained by the equal probability to reach the optimum solution in each generation. The evolution (number of generations in which the determination coefficient improved) proved to follow the Poisson distribution and could be related to the number of alive regressions in cultivar. The frequency distribution of the determination coefficient proved to be normal. Its range of difference between the values obtained by complete search and the GA-MLR was from 0.0090 to 0.0213. Note that the evolutions were not identical, even with constant optimization parameters.

The Steiger Z test was applied in order to identify the significances between the correlation coefficient of SPRs obtained by GA-MLR and the correlation coefficient obtained through complete search. The results revealed that the correlation coefficient of GA-MLR was not statistically different from the one obtained by complete search (Fig. 4).

The prediction ability of these GA-MLR models needs further reliability tests (ability checked in validation sets). Therefore, further research is needed. One of the...
main advantages of the GA is given by its ability to identify more models with high determination coefficients. Thus, for a sample of compounds, more models with similar performances in terms of the determination coefficient are available. Some of these models are able to characterize the relationship between structure and property by the same compound characteristics than the model identified through complete search (e.g., geometry, cardinality, etc.).

Conclusions. – The implemented GA was able to select two MDF descriptors able to explain the relationships between the structure of PCBs and lipophilicity. Thirty independent runs were investigated. The results showed that although each run generated a different optimum solution, these solutions were not statistically different from the solution obtained by complete search. The highest value of the determination coefficient, the minimum value of the sum of residuals in estimate, and the minimum value of entropy were the three criteria used to identify the best runs.

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Experimental Part

Compound Set and Complete Search. A sample of 206 polychlorinated biphenyls (PCBs) was studied [30]. The octan-1-ol/H\(_2\)O partition coefficient was the subject of SPR analyses; the experimental values were taken from previously published articles [31–39]. The generic structure of the compounds, abbreviations, and measured octan-1-ol/H\(_2\)O partition coefficients are available as Supplementary
Material). The experimental data proved to be normally distributed at a significance level of 5% (Kolmogorov–Smirnov statistic, 0.0335 (p = 0.9691); Chi-square statistic, 11 (p = 0.1386, df = 7), mean, 6.58; and standard deviation 0.83).

The MDF, comprising a total number of 787968 members, was applied [26] on PCBs and SPR models were obtained. The complete search for pairs of two descriptors (310446390528 candidate solutions) provided the following model:

\[
\hat{Y}_{\text{MDF-2D}} = 3.121(\pm 0.347) - 0.441(\pm 0.064) \cdot I D D K G g + 0.045(\pm 0.002) \cdot I H D R K E g
\]

\[
r = 0.9433 \quad (95\% \text{ CI } (0.9259 – 0.9566))
\]

\[
r^2 = 0.8897, s_{\text{est}} = 0.28, F_{\text{est}} (p_{\text{est}}) = 819 (6.36 \cdot 10^{-9})
\]

\[
r^2_{\text{cv-loo}} = 0.8854, s_{\text{loo}} = 0.28; F_{\text{pred}} (p_{\text{pred}}) = 784 (9.33 \cdot 10^{-9})
\]

where \(\hat{Y}_{\text{MDF-2D}}\) is the estimated octan-1-ol/H\(_2\)O partition coefficient (according to the MDF model with two descriptors), \(I D D K G g\) and \(I H D R K E g\) are molecular descriptors (members of MDF), and \(r\) is the correlation coefficient, \(r^2\) the determination coefficient, \(s_{\text{est}}\) the standard error of estimate, \(F_{\text{est}} (p_{\text{est}})\) the \(F\)-value and the associated probability, \(r^2_{\text{cv-loo}}\) the cross-validation leave-one-out score, \(s_{\text{loo}}\) the standard error of predicted, and \(F_{\text{pred}} (p_{\text{pred}})\) the \(F\)-value and the \(p\)-value in the leave-one-out analysis, resp.

**Genetic Algorithms (GA).** The search in the MDF pool for descriptors to be used in MLR for SPR could be regarded as a hard problem (HP). Two types of information are available for a sample of chemical compounds, a pool of molecular descriptors (structural information obtained from molecular topology and geometry-based models of quantum and molecular physics) and an observed property. Therefore, the question is: ‘which SPR is best able to describe the property as function of the compounds’ structure?’

The search for MLR with MDF is a HP because the search space increases exponentially with the increase in the number of descriptors. Moreover, the execution time is out of a real-time for MLR with more than three descriptors.

The implementation and use of a GA offer the advantage of a heuristic search, compared to a complete search that implies the exploration of all possible combinations to identify the MLR model.

The molecular structure of the PCBs was drawn using the HyperChem program [40], and the 3D geometry was optimized. Partial charges were calc. using the semi-empirical extended Hückel model (single point approach) [41], and the geometry of the compounds was optimized by applying the Austin method (AM1) [42]. The obtained outputs stored information on the topology, geometry, and charge distribution of the PCBs and served as primary data for generating the MDF [43].

The GA designed is described below (see Fig. 5):

- **Step 0 (search space):** definition of the genetic representation of the feature selection applied to select proper descriptors for the MLR problem of the property predicted by SPR.

- **Step 1 (initial sample):** generation (random selection) of the initial sample of the MDF members calc. for PCBs (‘Create tables’ and ‘Insert MDF and property’, see Fig. 5). This contains the candidate solutions. The genetic representation of the MDF was defined: a molecular descriptor represents a genotype described by the following genes:
  - ‘d’ Gene: encodes the distance operator and could take two values, ‘g’ for geometrical distance and ‘t’ for topological distance.
  - ‘p’ Gene: encodes the atomic property used to construct the phenotype and could take six values: ‘M’ (relative atomic mass), ‘Q’ (atomic partial charge, semi-empirical extended Hückel model, single point approach), ‘C’ (cardinality, trivial atomic property; its value for any atom is equal to 1), ‘E’ (atomic electronegativity, i.e., the relative value on the Sanderson electronegativity scale), ‘G’ (group electronegativity, i.e., the value obtained by calculating the geometric mean of electronegativity associated with the group of atoms that are neighbors of the investigated atom), and ‘H’ (number of H-atoms that are neighbors of the investigated atom).
  - ‘I’ Gene: encodes the interaction descriptor and could take one of the following 22 values (where ‘d’ is the distance operator and ‘p’ is the atomic property): ‘D(d)’, ‘d(1/d)’, ‘O(p)’, ‘o(1/p)’, ‘+’,
Fig. 5. Steps in the genetic algorithm selection of molecular descriptors family members for PCBs

'O' Gene: encodes the overlapping interactions. Six values were implemented, two for the models with sporadic and distant interactions ('R' and 'r', resp.), two for the models with frequent and distant interactions ('M' and 'm', resp.), and two for the models with frequent and closed interactions ('D' and 'd', resp.).

't' Gene: encodes the algorithm of molecular fragmentation on pairs of atoms and could take one of the following values: 'P' (fragmentation based on paths), 'D' (fragmentation based on distances), 'M' (fragmentation in maximal fragments), and 'm' (fragmentation in minimal fragments, trivial fragments with one atom).

'M' Gene: encodes the global overlapping of fragment interactions and could take one of the following 19 values classified into four groups: i) values' group ('m', minimum value; 'M', maximum value; 'n', lowest absolute value; 'N', highest absolute value), ii) means' group ('S', sum; 'A', arithmetic mean of the number of fragments' properties; 'a', arithmetic mean of the number of fragments; 'B', arithmetic mean of the number of atoms; 'b', arithmetic mean of the number of bonds), iii) geometrics' group ('P', multiplication; 'G', geometric mean of the number of fragments' properties; 'g', geometric mean of the number of fragments; 'F', geometric mean of the number of atoms; 'f', geometric mean of the number of bonds), and iv) harmonics' group ('s', harmonic sum; 'H', harmonic mean of the number of fragments' properties; 'h', harmonic mean of the number of fragments; 'I', harmonic mean of the number of atoms; 'i', harmonic mean of the number of bonds).

'L' Gene: encodes one of the following six linearization operators: 'I' (identity), 'i' (inverse), 'A' (absolute value), 'a' (inverse of absolute value), 'L' (logarithm of absolute value), and 'l' (logarithm). One of these operators is applied during the evaluation of the fittest for every descriptor of the sample.
The entire population is of 131,328 molecular descriptors (excluding the six above cited linearization operators from the multiplication). A small number are included in the sample, which contains a fixed number of genotypes. Eight were used for this experiment.

- Step 2 (adaptation): transformation of the genotypes into phenotypes by checking their values in the environment given by the experimental (measured) data and by applying the linearization operator. The following were applied to the adaptation of each phenotype:
  - For the minimum absolute variance (a ratio of measured data variance), 0.1 was used.
  - For the maximum Jarque–Bera value (no higher than the value of a Jarque–Bera ratio on the measured data \[44\]), 1.0 was used.
  - For the minimum determination coefficient with experimental data (higher than a ratio), 0.1 was used.

- Step 3 (fittest): the fittest score of an individual was defined as the minimum determination coefficient obtained in MLR with all the other individuals in the sample.

- Step 4 (phenotyping): the fittest score of an individual can be defined using different expressions; every expression characterizes the individual in one way; a series of other fittest scores are calculated (as given in Table 4) for analysis purpose and an output is given for each generation.

### Table 4. Scores for the Genetic Algorithms

<table>
<thead>
<tr>
<th>Score ((i = 1..2))</th>
<th>Significance</th>
<th>Objective</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_e = \sum (Y_i - \hat{Y}_i)^2)</td>
<td>Sum of residuals in the estimate ((S_e))</td>
<td>Minimum (p^*)</td>
<td>(p^*) is frequent equal with 1.0</td>
</tr>
<tr>
<td>(r^2 = (r^2(\hat{Y}, \hat{Y}^2))^p)</td>
<td>Coefficient of determination ((r^2))</td>
<td>Maximum (\hat{Y} = b_0 + \sum b_i \cdot \text{Phen}_i); (p) frequent equal with 2.0</td>
<td></td>
</tr>
<tr>
<td>(tr = \min(t_i))</td>
<td>Geometric mean of evolution ((tr))</td>
<td>Maximum (\hat{Y} = b_0 + \sum b_i \cdot \text{Phen}_i); (t_i =</td>
<td>t(b_i)</td>
</tr>
<tr>
<td>(Hr = H(r^2, 1 - r^2; p))</td>
<td>Entropy of determination or undetermination event ((Hr))</td>
<td>Minimum</td>
<td>It takes a value of 1 (maximum) when the determination is maximum or when the undetermination is minimum. It takes a value of 0 (minimum) when the determination is equal to the undetermination ((r^2 = 0.5)).</td>
</tr>
</tbody>
</table>

*) \(p\) \(=\) a constant defined by the user. \(b) t_i = \text{Student’s } t\)-parameter associated to the coefficients of regression.

- Step 5 (selection): selects pairs of individuals from the sample for reproduction. The proportional selection method is used (the frequencies of selection are proportional to the fittest scores). The selected individuals are subjects of genotype crossover and mutation. The equation used for the selection was \(p_i = f_i/\sum f_i\) (where \(p_i\) is the probability used for the selection, \(f_i\) = fittest score).

- Step 6 (crossover and mutation): crossovers the selected individuals to generate offspring. If the genotypes are given by: Genotype\(_1\) = \(d_1p_1I_1O_1f_1M_1\) and Genotype\(_2\) = \(d_1p_1I_1O_1f_1M_2\), then two numbers (acting as crossover boundaries) are randomly generated within the 0 to 5 range (e.g., 2 and 4), and the crossover genotypes generate offspring (e.g., Child\(_1\) = \(d_1p_1I_1O_1f_1M_1\) and Child\(_2\) = \(d_1p_1I_1O_1f_1M_2\)). If a mutation is decided (with a low probability: 0.05 was used), one of two individuals is chosen (randomly) and the mutation is applied. Mutation implies the random selection of a gene that will be mutated and the random mutation of that gene.

- Step 7 (survival): offspring replace two individuals in the sample in the following order: dead, parents, others. At the end of this step, an evolution cycle is complete and a new generation of the sample is generated.
Step 8 (evolution): the GA continues with Step 2 again, unless a number of generations was exhausted (1000 was used) or an imposed value of the best (or worst) fittest score was obtained.

The objective of the GA was to obtain the MLR with two MDF members having the highest determination coefficient. The GA was implemented as Windows based FreePascal application with MySQL connectivity for fetching the data. The application was run 30 times on PCBs, to assess the algorithm performance in terms of speed and its ability to identify the optimum solution. The imposed maximum number of generations was equal to 1000. The optimization criterion used in this search was to maximize the minimum value of determination coefficient obtained from GA-MLR.

The Steiger’s Z test was used to compare the GA-MLR correlation coefficient obtained with that identified in the complete search (H₀ hypothesis: the correlation coefficient obtained in GA-MLR is not different from the correlation coefficient obtained in the complete search) [45]. The Z critical value for a significance level of 5% was equal to 1.96 (Z₀c ∈ (–∞, 1.96] ∪ [1.96, +∞), then the H₀ is rejected). Statistica 8.0 was used to investigate the type of distribution on each investigated criterion.

REFERENCES

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